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FLOW OF A POLYMER SOLUTION BETWEEN TWO ROTATING ROLLS

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A new model of calendering the film of a viscoelastic polymer solution based on the principles of mechanics of saturated porous media is suggested. The point of exit of the film from the gap is not prescribed as is done in traditional approaches but is determined from the condition of adhesion strength of the contact of the polymer with the rolls. An analytical solution of the stationary problem is obtained. Disturbances of this solution caused by periodic changes in the thickness of an incoming flow or a pulling rate are investigated.

Introduction. The traditional mathematical models of calendering are based on hydrodynamics of non-Newtonian fluids in the gap between rolls and are intended for determination of the thrust forces acting onto the rolls and a change in the film thickness and in the material structure [1–5]. The common drawback of such models is the indeterminacy of the point of exit of a polymer from the gap (the exit point). This difficulty is overcome, as a rule, by two methods. In the first method, the thickness of an outgoing film is considered to be known from experiment [1, 3]. In the second, at the exit point an additional condition of the local extremum of pressure is adopted [1, 2, 4–6]. Below, it will be shown that this requirement does not provide, generally speaking, an actual value of the outlet thickness of the film.

In the present work, we suggest a new model of considering a polymer solution that is based on the principles of mechanics of saturated porous media [7, 8]. The permolecular structures of the polymer play the role of a porous matrix whose rheological behavior is described by the Oldroyd viscoelastic model [9]. The motion of a solvent serving as a saturating fluid relative to the polymer is determined by the law of filtration in a deformable porous medium [8]. The point of exit of the film from the gap is determined in solving the problem from the condition of the adhesion strength of contact of the polymer with the surface of rolls.

An analytical solution of the stationary problem is constructed. Distributions of the solvent pressure and of the stresses in the polymer are found. The condition of implementation of the cavitation-free regime is obtained when the solvent pressure in the gap does not reduce lower than the pressure of its saturating vapor i.e., no nucleation occurs in the film.

We have also set and solved problems on disturbances of the steady state caused by periodic changes in the inlet thickness of the film or in the pulling rate. The dependences of the amplitudes of change in the stress in the polymer, of the solvent pressure, and of the thrust force on the frequency are found.

1. Mathematical Formulation of the Problem. The Basic Equations. A scheme of film flow of the polymer solution in a calender gap is given in Fig. 1.

It is assumed that the flow is two-dimensional and the radii R of the cylindrical rolls are equal; the rate of material pulling v is prescribed. The x-axis is in the plane of symmetry of the gap and is extended in the direction of film flow, and the y-axis runs through the axes of the rolls. The basic geometric condition is

$$h << a << R . \tag{1}$$

This means that the region is narrow ($h << R$) and long ($a >> h$). Here, the deformations of the polymer matrix can be considered to be small and transverse (y), while the filtration of the solvent (with allowance for impenenetrability of the rolls) can be considered longitudinal (x). The narrow transient region, in which the motion of a free film is reorganized into the flow in the gap, will be modeled by the line (point) of entry $x = -a$, where the film profile under-

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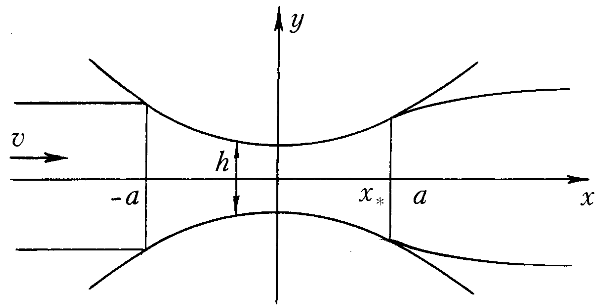


Fig. 1. Scheme of the motion of the polymer solution film in the calender gap.

goes bending. By virtue of (1) all kinematic characteristics of the polymer are known and determined by the film velocity and the form of the gap:

$$h(x) = h_0 + \frac{x^2}{R}, \quad -a < x < a. \tag{2}$$

Thus, deformations of the polymer matrix in stationary motion are as follows:

$$\varepsilon_x = \varepsilon_z = 0, \quad \varepsilon_y = \varepsilon = \frac{h(x) - h(-a)}{h(-a)} = \frac{x^2 - a^2}{Rh_0 + a^2}. \tag{3}$$

The rate and acceleration of deformation are characterized by the quantities

$$e(x) = \frac{d\varepsilon}{dt} \equiv \dot{\varepsilon} = v\frac{\partial \varepsilon}{\partial x} = \frac{2vx}{Rh_0 + a^2}, \quad \dot{e}(x) = \frac{2v^2}{Rh_0 + a^2}. \tag{4}$$

The process of filtration in a deformable porous medium is described by the continuity equation and the filtration law. For the conditions considered, in conformity with [8] we have

$$e + \frac{dq}{dx} = 0, \quad q = -\frac{k(\varepsilon)}{\mu}\frac{dp}{dx}. \tag{5}$$

For small deformations we assume that $k(\varepsilon) \simeq k(0) = k$. From (5) we obtain the following equation:

$$\frac{d^2p}{dx^2} = \frac{\mu e(x)}{k}, \quad -a < x < x_*, \tag{6}$$

describing the distribution $p(x)$ in the gap; x_* is the unknown coordinate of exit of the film.

As a rheological model of the porous matrix, we adopt the Oldroyd model [9]

$$\sigma + \lambda\dot{\sigma} = \mu_0(e + \tau\dot{e}), \tag{7}$$

where the overdot indicates the material time derivative and σ is the effective stress [7] of fibers parallel to the y-axis. The physical meaning of this quantity is reflected in the formula for the thrust force:

$$F = \int_{-a}^{x_*}(p - \sigma)\,dx. \tag{8}$$

For stationary motion, Eq. (7) with account for (4) acquires the form

$$\sigma + \lambda v \frac{d\sigma}{dx} = \mu_0 \left(e + \tau v \frac{de}{dx} \right). \tag{9}$$

Relations (6) and (9) are the basic equations of the model in a stationary case.

Let us pass to formulation of the boundary conditions. For pressure p we have

$$p(-a) = p(x_*) = p_a\,, \tag{10}$$

where p_a is atmospheric pressure.

The inlet values of σ are obtained from (9) in the following way. Within the framework of the adopted scheme of flow, the functions $e(x)$ and $\dot{e}(x)$ can be extended to the left beyond the point $x = -a$ and represented in the form

$$e(x) = H(x + a) \frac{2vx}{Rh_0 + a^2}\,; \quad \dot{e} = \delta(x + a) \frac{2av^2}{Rh_0 + a^2}\,,$$

where $H(x)$ and $\delta(x)$ are the Heaviside and Dirac functions.

Integrating (9) over $-a - \Delta < x < -a + \Delta$ at $\Delta \to 0$, we will have

$$\sigma(-a + 0) = \sigma_0 = -\frac{Nv\mu_0\tau}{a\lambda}\,, \tag{11}$$

where $N = 2a^2/(a^2 + Rh_0)$.

Unlike the traditional models [16], in the suggested approach the position of the exit point x_* is determined in the course of solving the problem from the condition of adhesion strength to separation of the contact of the polymer with the surface of rolls: $\sigma(x_*) = \sigma_* \ge 0$.

Next, for the sake of simplicity we confine ourselves to a special case:

$$\sigma(x_*) = 0\,. \tag{12}$$

Thus, the problem consists of the determination of $\sigma(x)$, x_*, and $p(x)$ from (6), (9)–(12) by the prescribed geometric (h_0, a, R), rheological (τ, λ, μ_0), filtration (k, μ), and operational (v, σ_*) parameters.

2. Solution of the Stationary Problem. With account for (4), Eq. (9) acquires the form

$$\sigma + \lambda v \frac{d\sigma}{dx} = \mu_0 N v \frac{x + v\tau}{a^2}\,.$$

Hereinafter we will scale x on a, σ on $\mu_0 v/a$, and λ and τ on a/v. In the new scales, we have the following equation for σ:

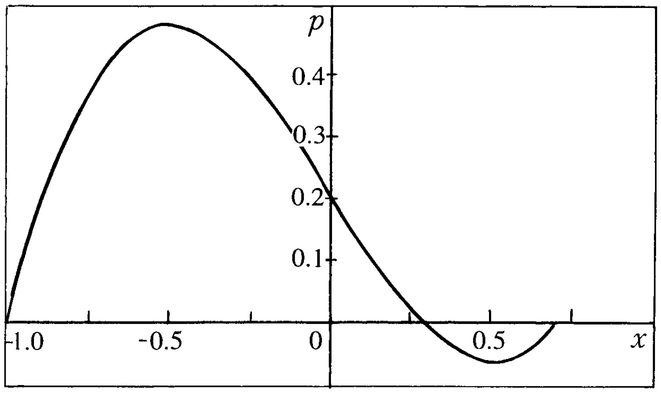
$$\sigma + \lambda \frac{d\sigma}{dx} = N(x + \tau)$$

and the boundary condition

$$\sigma(-1) = -\frac{N\tau}{\lambda}\,,$$

whence

$$\frac{\sigma}{N} = x + \tau - \lambda + (\lambda - \tau)\left(1 + \frac{1}{\lambda}\right)\exp\left(-\frac{1 + x}{\lambda}\right) = f(x, \tau, \lambda)\,. \tag{13}$$



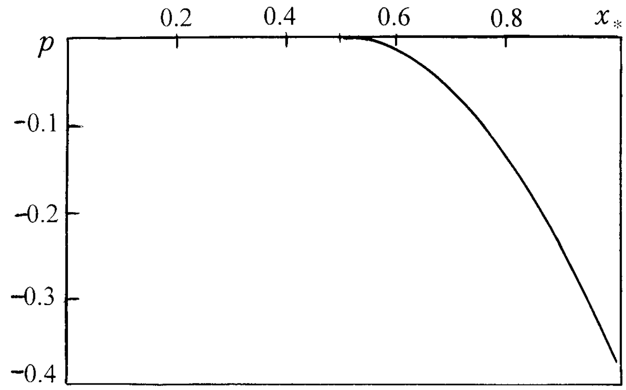


Fig. 2. Pressure distribution in the gap.

Fig. 3. Minimum pressure versus the coordinate of the point of exit of the film from the gap.

Now condition (12) allows determination of the exit point x_* as a root of the equation $f(x, \tau, \lambda) = 0$. We can make sure that in the case of a purely viscous polymer matrix, $x_* = 0$, and in the case of an elastic matrix, $x_* = 1$. In the general case, $0 < x_* < 1$.

Having determined the point x_* of exit of the polymer film from a working section of the gap, we will find the pressure distribution of the solvent. We will reckon this pressure from the atmospheric one and normalize to $va\mu/k$. As a result, Eq. (6) and boundary conditions (10) acquire the form

$$p_{xx} = Nx\,, \quad p\,(-1) = p\,(x_*) = 0\,,$$

whence

$$p = \frac{N}{6}\,(x^3 - x\,(x_*^2 - x_* + 1) - x_*^2 + x_*)\,. \tag{14}$$

As mentioned above, one of the traditional methods of determination of the exit point is the additional condition of minimum pressure [1, 2, 4–6]. We will show that this conditions is, generally speaking, ill-defined. By virtue of (14) we have

$$\left.\frac{dp}{dx}\right|_{x=x_*} = G = \frac{N}{6}\,(2x_*^2 + x_* - 1)\,.$$

At $0 < x_* < 1/2$, the value of $G < 0$ and $p(x)$ in a working section of the gap $-1 < x < x_*$ is positive. At $1/2 < x_* \le 1$, $G > 0$ and in the neighborhood of the exit there is a zone of negative pressures. And only at $x_* = 1/2$ is the traditional condition

$$\left.\frac{dp}{dx}\right|_{x=x_*} = 0$$

fulfilled. The typical distribution $p(x)$ and the dependence of minimum pressure on x_* are represented in Figs. 2 and 3.

It should be noted that the presence of negative (i.e., lower than atmospheric) pressures in the working section of the gap can lead to boiling-up of the solvent and nucleation in the film. In this connection, it is reasonable to determine the conditions of implementation of the cavitation-free regime of flow:

$$p\,(x) > p_0\,(T)\,, \quad -1 \le x \le x_*\,, \tag{15}$$

where $p_0(T)$ is the pressure of the saturated vapor of the solvent. Since the minimum pressure $p(x)$ is determined by the position of the point x_* (Fig. 3), there the dependence $x_*(p_0)$ exists and inequality (15) is equivalent to the limitation $0 < x_* < x_*(p_0)$. In the special case $p_0 = 0$, we have $0 < x_0 < 1/2$.

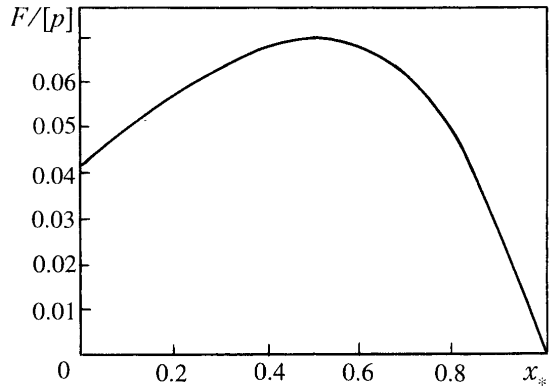


Fig. 4. Dimensionless value of the thrust force as a function of the point of exit of the film from the gap, $[p] = va\mu/k$.

With the aid of (8), (13), and (14) we can find the thrust force F. Here, it should be taken into account that the characteristic scales of pressure $[p]$ and stress $[\sigma]$ are substantially different: $[p] = va\mu/k$, $[\sigma] = \mu_0 v/a$. Therefore,

$$\frac{F}{[p]} = \int_{-1}^{x_*} \left(p - \frac{[\sigma]}{[p]} \sigma \right) dx .$$

To evaluate $[p]/[\sigma] = a^2\mu/(k\mu_0)$, we assume that $\mu_0 = 100\mu$, $k = 10^{-12}$ m^2, and $a = 10^{-1}$m. Then $[p]/[\sigma] = 10^8$. This means that the thrust force is determined primarily by pressure and can be calculated by the formula

$$\frac{F}{[p]} = \int_{-1}^{x_*} p dx = -\frac{x_*^4}{24} - \frac{x_*^3}{12} + \frac{x_*}{12} + \frac{1}{24} . \tag{16}$$

A plot of the dependence $(F/[p])(x_*)$ is presented in Fig. 4.

Note that the outlet thickness of the film $h_* = h(x_*)$ is, generally speaking, not retained in the free state. By virtue of (4) and (7) its change is determined by the equation $e + \tau v \frac{\partial e}{\partial x} = 0$ and boundary condition $e(x_*) = \frac{Nvx_*}{a^2}$. Whence

$$e = v \frac{d\varepsilon}{dx} = \frac{Nx_*}{a^2} \exp\left(\frac{x_* - x}{v\tau} \right).$$

Integrating this expression over the interval $[x_*, \infty)$, we find the quantity $\varepsilon(\infty) - \varepsilon(x_*) = Nx_*\tau v/a^2$ that characterizes, in conformity with (3), the change in the thickness of a film after escape from the gap.

In actual practice, the initial parameters of the problem can experience deviations of various kind from stationary values. Therefore, it is interesting to investigate disturbances of the steady-state solution due to such deviations.

3. Response of a Solution of the Problem to a Change in the Inlet Thickness of a Film. In this case, an important additional feature appears — along with the film thickness the coordinate of the point of entry of the film into the gap (the entry point) becomes variable. On the basis of Fourier analysis, it is sufficient to consider the case of harmonic disturbances; therefore, the coordinate of the entry point x_0 will be prescribed as $x_0 = -a(1 + \alpha \exp(i\omega t))$, where $0 < \alpha << 1$.

Deformations of the polymer matrix are determined, as previously, by the form of the gap and described by the relation

$$\varepsilon(x, t) = \frac{h(x) - h(x_0(t'))}{h(x_0(t'))} .$$

Here, it is necessary to take into consideration that the instants of time t and t' are related by the law of motion of a material element of the film: $x = x_0 + v(t - t')$. Otherwise, the element considered at the point x is at the entry point at the instant of time $t' = t - (x - x_0)/v$. Thus

$$\varepsilon (x, t) = \frac{h (x) - h \left(- a - \alpha\, a \exp \left(iw \left(t + \frac{x_0 - x}{v} \right) \right) \right)}{h \left(- a - \alpha\, a \exp \left(iw \left(t + \frac{x_0 - x}{v} \right) \right) \right)} . \tag{17}$$

Bearing in mind that the disturbances are small, we assume that $\varepsilon(x, t) = \bar{\varepsilon}(x) + \alpha\varepsilon'(x, t)$ where $\bar{\varepsilon}(x)$ is determined by formula (3) while $\varepsilon'(x, t)$ are found by performing linearization of (17)

$$\varepsilon' (x, t) = - 2a^2 \exp \left(iw \left(t - \frac{a + x}{v} \right) \right) \frac{Rh_0 + x^2}{(Rh_0 + a^2)^2} . \tag{18}$$

The rate and acceleration of deformation are characterized, respectively, by the quantities

$$e' (x, t) = \frac{d\varepsilon'}{dt} = - 2a^2 \exp \left(iw \left(t - \frac{a + x}{v} \right) \right) \frac{2vx}{(Rh_0 + a^2)^2} H (x - x_0) ,$$

$$\frac{de'}{dt} = - 2a^2 \exp \left(iw \left(t - \frac{a + x}{v} \right) \right) \frac{2v^2}{(Rh_0 + a^2)^2} (H (x - x_0) + x\delta (x - x_0)) . \tag{19}$$

Here the functions H and σ serve, as previously, for including the entire neighborhood of the entry point into the domain of definition of kinematic characteristics which is used in deriving the boundary conditions for stresses.

Let us represent stress in the form $\sigma = \bar{\sigma} + \alpha\sigma'$, where $\bar{\sigma}(x)$ is given by expression (13) and $\alpha\sigma'(x, t)$ is the disturbance. Then rheological equation (7) acquires the form

$$\bar{\sigma} + \alpha\sigma' + \lambda\alpha \frac{\partial\sigma'}{\partial t} + \lambda v \frac{\partial\bar{\sigma}}{\partial x} + \lambda v\alpha \frac{\partial\sigma'}{\partial x} = \mu_0 (e + \tau\dot{e}) . \tag{20}$$

An equation for $\sigma'(x, t)$ in the domain $-x_0 < x < x_*$, $t > 0$ follows from (19) and (20):

$$\sigma' + \lambda \frac{\partial\sigma'}{\partial t} + \lambda v \frac{\partial\sigma'}{\partial x} = - 4va^2\mu_0 \exp \left(iw \left(t - \frac{a + x}{v} \right) \right) \frac{x + v\tau}{(Rh_0 + a^2)^2} . \tag{21}$$

We will seek σ' in the form

$$\sigma' (x, t) = f (x) \exp (iwt) , \tag{22}$$

where $f(x)$ is some complex function. Then from (21) we find the equation

$$(1 + \lambda iw) f (x) + \lambda v \frac{\partial f}{\partial x} = - 4a^2 v\mu_0 \exp \left(- iw \frac{a + x}{v} \right) \frac{x + v\tau}{(Rh_0 + a^2)^2} . \tag{23}$$

The boundary condition for stress is obtained by integrating (2) in the neighborhood of the entry point:

$$\sigma\,(x_0\,(t)) = \frac{\mu_0 \tau}{\lambda}\, e\,(x_0\,(t))\,. \tag{24}$$

Note that (24) is interesting in its universal nature independently of whether the entry point is fixed or not. For convenience we will transpose the boundary condition from the point $x_0 = -a(1 + \alpha \exp\,(i\omega t))$ to the point $x = -a$. Such an operation is equivalent to expansion of (24) into a Taylor series in terms of α in the neighborhood $\alpha = 0$.

With account for (4), (13), and (19), we obtain

$$\sigma'\,(-a) = \frac{\mu_0 \tau}{\lambda}\left(-\frac{2va\exp\,(iwt)}{Rh_0 + a^2} + \frac{4a^3 v \exp\,(iwt)}{(Rh_0 + a^2)^2}\right) + aN\exp\,(iwt)\left(1 + (\lambda - \tau)\left(1 + \frac{1}{\lambda}\right)\frac{1}{\lambda}\exp\left(-\frac{1-a}{\lambda}\right)\right).$$

Using representation (22), we find the sought boundary condition for $f(x)$ in the form

$$f\,(-a) = \frac{\mu_0 \tau}{\lambda}\left(\frac{4a^3 v}{(Rh_0 + a^2)^2} - \frac{2va}{Rh_0 + a^2}\right) + aN\left(1 + (\lambda - \tau)\left(1 + \frac{1}{\lambda}\right)\frac{1}{\lambda}\exp\left(-\frac{1-a}{\lambda}\right)\right).$$

Scaling f on $v\mu_0/a$, we will have the boundary-value problem for $f(x)$ in the previous dimensionless variables:

$$\frac{\partial f\,(x)}{\partial x} + \frac{\left(1 + \frac{iwa\lambda}{v}\right) f\,(x)}{\lambda} = -\frac{N^2\,(x + \tau)}{\lambda}\exp\left(-iw\,\frac{1+x}{v}\right), \tag{25}$$

$$f\,(-1) = \frac{\tau}{\lambda}\,(N^2 - N) + N\left(1 + \frac{1}{\lambda}\,(\lambda - \tau)\left(1 + \frac{1}{\lambda}\right)\right). \tag{26}$$

Having solved it, with account for (22) we find

$$\begin{aligned} \exp\,(-iwt)\,\sigma'\,(x, t) &= -N^2\,(x + \tau - \lambda)\exp\left(iw\,\frac{1+x}{v}\right) + \\ &+ \left[\frac{\tau}{\lambda}\,(N^2 - N) + N - (\lambda - \tau)\,(1 + \lambda)\,N\right]\exp\left(-\frac{x}{\lambda}\left(1 + iw\,\frac{a\lambda}{v}\right)\right) + \\ &+ N^2\,(-1 + \tau - \lambda)\exp\left(-\frac{1+x}{\lambda}\right)\exp\left(-iw\,\frac{1+x}{v}\right) = C\,(x) + iD\,(x)\,. \end{aligned} \tag{27}$$

Separating the real part of (27), we obtain

$$\sigma'\,(x, t) = A\,(x)\cos\,(wt - \varphi)\,,$$

where $A = \sqrt{C^2 + D^2}$ is the amplitude of disturbances of σ and $\varphi(x) = \arccos\left(\frac{C}{\sqrt{C^2 + D^2}}\right)$ is the phase shift.

To calculate corrections for the solvent pressure, it is necessary to know the displacement of the exit point x_*. Let $x_*(t) = \bar{x}_* + \alpha y(t)$. By virtue of (12)

$$\bar{\sigma}\,(\bar{x}_* + \alpha y) + \alpha\sigma'\,(x_* + \alpha y) = 0\,.$$

Whence in the linear, with respect to α, approximation

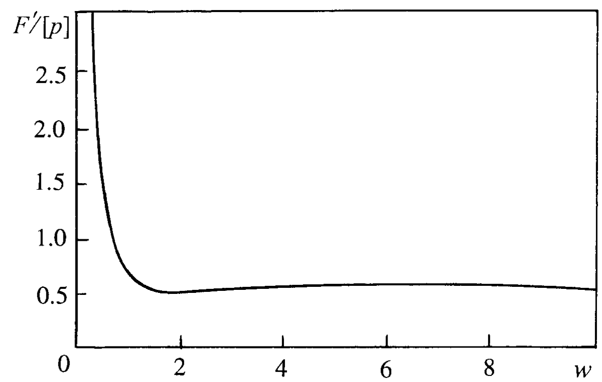


Fig. 5. Dependence of the correction for the amplitude of the thrust force on the frequency of disturbances of the thickness of the incoming film.

$$y(t) = -\frac{\sigma'(\bar{x}_*, t)}{\left.\frac{d\bar{\sigma}}{dx}\right|_{x=\bar{x}_*}} \equiv -\frac{\sigma'}{Z}\,. \tag{28}$$

Substituting the expansion $p(x, t) = \bar{p}(x) + \alpha p'(x, t)$ in (6) and (10) and accounting for (19), we will obtain

$$\frac{d^2 p'}{dx^2} = -\frac{4\mu a^2}{k\,(Rh_0 + a^2)^2} \exp\left(iw\left(t - \frac{a+x}{v}\right)\right).$$

Normalizing, as previously, the pressure to $va\mu/k$, we finally arrive at

$$p'_{xx} = -N^2 x \exp\left(iw\left(t - \frac{a+x}{v}\right)\right). \tag{29}$$

Now we rewrite conditions (10) as follows:

$$p\,(-1 - \alpha \exp(iwt)) = p\,(\bar{x}_* + \alpha y(t)) = 0\,. \tag{30}$$

whence

$$p'(-1) = \frac{N}{6}(-x_*^2 + x_* + 2)\exp(iwt)\,, \quad p'(x_*) = -\frac{N}{6}(2x_*^2 + x_* - 1)\,y(t)\,. \tag{31}$$

From (29) and (31) we find a correction for the pressure:

$$\begin{aligned}\exp(-iwt)\,p'(x, t) = N^2\left(\frac{xv^2}{w^2} - i\,\frac{2v^3}{w^3}\right)\exp\left(-iw\,\frac{1+x}{v}\right) - \\ - N^2\left(\frac{v^2}{w^2} + i\,\frac{2v^3}{w^3}\right) - N\,\frac{x}{6}(-x_*^2 + x_* + 2) + Q\,\frac{x+1}{x_*+1}\,,\end{aligned} \tag{32}$$

where

$$Q = -N^2\left(\frac{x_* v^2}{w^2} - i\,\frac{2v^3}{w^3}\right)\exp\left(-iw\,\frac{1+x_*}{v}\right) + x_* N^2\left(\frac{v^2}{w^2} - i\,\frac{2v^3}{w^3}\right) -$$

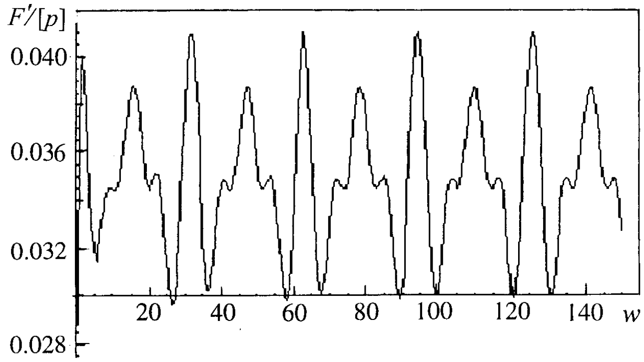


Fig. 6. Dependence of the correction for the amplitude of the thrust force on the frequency of disturbances of the rolling rate.

$$-\frac{Nx_*}{6}(-x_*^2+x_*+2)-N(2x_*^2+x_*-1)\frac{C(x)+iD(x)}{6Z};$$

C, D, and Z are determined in (27) and (28).

Now with account for formula (16) we can find a correction for the thrust force. A plot of the dependence of this correction on the frequency is given in Fig. 5. It is indicative of the absence of resonance frequencies.

Disturbance of the Steady-State Solution of the Problem Due to a Change in the Pulling Rate. Let us assume that the thickness of the inlet film is constant. The change in the pulling rate according to relations (4) entails changes in the deformation rate and its acceleration and, consequently, changes in the polymer stresses and in the coordinate of the exit point. As a result, the pressure distribution of the solvent pressure and the thrust force change as well. A solution of the problem within the framework of the linear theory of disturbances can be obtained with use of the technique described in Sec. 3. Thus, disturbance of the pressure has the form

$$p'(x,t)=\frac{N}{6}\exp(iwt)\left(x^3-x-\frac{(2x_*^2+x_*-1)(x+1)y(t)}{x_*+1}+\frac{-xx_*^3+xx_*-x_*^3+x_*}{x_*+1}\right), \tag{33}$$

where x_* is found from (12) and (13).

Figure 6 presents the corresponding dependence of the amplitude of disturbances of the thrust force on the frequency.

5. Results and Discussion. The suggested model of calendering is intended for polymer solutions when a medium as a whole can be represented as a system of two interacting continua with substantially different rheological properties. At the same time, the condition of the adhesion strength of contact of the polymer with rolls $\sigma(x_*)=\sigma_*\ge 0$ determining the position of the point of exit of the film from the gap in solving the problem is of general nature and can be employed for any media. It should be noted that the stresses σ in the polymer and the solvent pressure p make radically different contributions to the thrust force. In essence, this is a consequence of the approximation of the long narrow gap. Here, the deformations ε and along with them the stresses σ in the polymer are small. At the same time, the solvent pressure p provides redistribution of the material along the gap so that at actual values of the parameters v, a, μ, and k its characteristic value $(va\mu/k)$ appears to be substantial.

Note that the influence of σ on p is manifested not only in determination of the exit coordinate but also in the form of the boundary conditions for disturbance p'.

We hope that this model can be of help in solving other problems of processing of materials.

NOTATION

v, rolling rate of the material; R, radius of the cylindrical rolls; T, temperature; $2h$, gap thickness; h_0, minimum value of h; $2a$, gap length; ε, e, and $\dot{e}$, deformation, rate, and acceleration of deformations of the polymer matrix; p and μ, pressure and viscosity of the solvent; q, filtration rate of the solvent relative to the polymer matrix; k, penetrability of the permolecular polymer structures; F, thrust force; σ, effective stress; λ, τ, and μ_0, rheological parameters; x_*, coordinate of the point of exit of the film from the gap; H and δ, Heaviside and Dirac functions; σ_*, limit of the adhesion strength of contact; t, time; ω, frequency of disturbances; α, small parameter; $x_0(t)$, coordinate of the entry point; N, geometric parameter; σ', ε', F', and $y(t)$, corrections for the effective stress, the deformation, the thrust force, and for the coordinate of the exit point; A, amplitude of the disturbances of σ; φ, phase shift.

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